J. Fluid Mech. (1978), *vol.* 85, *part* 3, *pp.* 591–606
Printed in Great Britain

A universal Strouhal number for the 'locking-on' of vortex shedding to the vibrations of bluff cylinders

By OWEN M. GRIFFIN

Ocean Technology Division, Naval Research Laboratory, Washington D.C. 20375

(Received 16 May 1977)

It is well known that the vortices shed from a circular cylinder lock on in frequency to the vibrations when the cylinder is forced to vibrate or is naturally excited to sufficient amplitudes by flow-induced forces. This paper presents a model for a universal wake Strouhal number, valid in the subcritical range of Reynolds numbers, for both forced and vortex-excited oscillations in the locking-on regime. The Strouhal numbers thus obtained are constant at $St^* = 0{\cdot}178$ over the range of wake Reynolds numbers $Re^* = 700\text{–}5 \times 10^4$. This value is in good agreement with the results obtained by Roshko (1954*a*) and Bearman (1967) for stationary circular cylinders and other bluff bodies in the same range of Reynolds numbers. A correspondence between the amplification of the cylinder base pressure, drag and vortex circulation is demonstrated over a wide range of frequencies and for vibration amplitudes up to a full cylinder diameter (peak to peak). The fraction ϵ of the shed vorticity in the individual vortices is found to be dependent upon the base-pressure parameter $K = (1 - C_{pb})^{\frac{1}{2}}$. Consequently, ϵ is also a function of the amplitude and frequency of the vibrations in the locking-on regime.

1. Introduction

If a fluid is in relative motion past a stationary bluff cylinder and the vortex-shedding frequency approaches one of the natural frequencies of the structure, then resonant flow-induced oscillations of the cylinder often occur when the damping of the system is sufficiently low. These resonant oscillations are accompanied by a 'locking-on' or capture of the vortex-shedding frequency by the vibration frequency over a range of flow speeds. This phenomenon is also encountered when a bluff body is forced to vibrate over a range of frequencies near the Strouhal frequency which is characteristic of the flow. The periodic lift and the mean drag forces are amplified as a result of these vibrations, and changes in these fluid forces are closely related to changes in the flow field in the near wake of the body.

The purpose of this paper is to examine the near wakes of cylinders vibrating laterally relative to an incident flow, and to relate the changes in certain characteristics of the wake flow to the amplification of the fluid forces acting on the body. A universal Strouhal number St^* is devised which is based upon the natural Strouhal frequency f_s, the mean velocity U_b at the edge of the boundary layer and wake width d' at the end of the vortex formation, viz. $St^* = f_s d'/U_b$. This formulation is found to hold equally well for both forced vibrations and resonant flow-induced vibrations, thus further confirming the equivalence of the two forms of resonant wake–body interaction first demonstrated by Griffin (1972). Both the velocity U_b and the wake width d' are found from experiment to be functions of the amplitude and frequency of vibration. Once the velocity U_b and the base-pressure coefficient $-C_{pb}$ have been found from experiment, it is possible to show that the fraction ϵ of shed vorticity in the fully developed vortices is an increasing function of the base-pressure parameter K, where $K = (1-C_{pb})^{\frac{1}{2}}$. The latter parameter is a function of the amplitude and frequency of vibration, so that the fraction of the shed vorticity in the fully formed vortices is similarly influenced by the resonant wake–structure interaction.

2. Related investigations

A number of survey papers which treat various aspects of unsteady flow phenomena and, in particular, the vortex-excited oscillations of bluff bodies and the equivalent forced vibrations have recently appeared. These include the proceedings of an international symposium on flow-induced structural vibrations (Naudascher 1974), which contain papers ranging from basic laboratory and computer studies of vortex formation to field studies of the vibrations of offshore structures. Research in unsteady flow phenomena during the early 1970's was reviewed by Berger & Wille (1972) and that up to 1976 by McCroskey (1977). Particular attention was paid in these papers to the locking-on between vortex shedding and a vibrating bluff body, no doubt because of both the practical and the fundamental interest in this phenomenon.

Some years ago Roshko (1954*a*, 1955) formulated a universal Strouhal number to scale the complex flow separation and vortex formation processes which take place in the near wakes of stationary bluff bodies of various cross-sections. Concurrently he proposed a semi-empirical model (1954*b*) for joining the outer potential flow to the wake. More recently Bearman (1967) expanded on Roshko's original ideas and proposed a somewhat different semi-empirical model for unifying the various parameters which affect the vortex formation and wake dynamics. The two formulations were basically similar in that the wake width, the primary length scale in the two models, was determined analytically. In the earlier model the wake width was determined from a 'notched-hodograph' theory (Roshko 1954*b*), while in the later formulation the so-called Kronauer stability criterion was employed in order to compute the wake width (Bearman 1967). The vortex-shedding frequency and the base pressure or, equivalently, the mean velocity on the free streamline at separation were obtained from experimental measurements in both cases. As a result, Bearman devised a universal Strouhal number St^* which collapsed the wakes of stationary bluff bodies of various shapes onto a constant value of $St^* = 0{\cdot}181$. The validity of the concept of a universal Strouhal number was recently extended to cylinders in confined flow by Richter & Naudascher (1976).

Most of the papers which deal with fundamental studies of forced and vortex-excited oscillations have appeared in the last ten years. For example, Koopmann (1967) measured the boundaries of the locking-on for forced vibration of a cylinder and photographed the resulting wake patterns for a variety of frequencies and amplitudes of vibration. The effects of cylinder vibration on the spanwise coherence of the vortex shedding and on the near-wake velocity fluctuations were measured by Toebes (1969). Griffin (1971) investigated the changes in vortex formation and the wake configuration which occur as a result of the locking-on for Reynolds numbers up to 350, and Griffin & Votaw (1972) extended these findings with measurements of the vortex spacing, velocity fluctuations and mean flow for various conditions of locking-on. Griffin & Ramberg (1974, 1975) measured the effects of the vibration amplitude and frequency on the vortex strength, spacing and drag at low Reynolds numbers and related these parameters to changes in the near-wake flow and in the vortex formation region. The drag forces which accompany the locking-on during vortex-excited oscillations were measured by Griffin, Skop & Koopmann (1973) and increases in the drag of up to 80 % above the stationary-cylinder value were found at large vibration amplitudes at Reynolds numbers of 600–800. Direct measurements of changes in the base pressure during locking-on were made by Stansby (1976), who observed that the increase in the base suction at Reynolds numbers near 5500 and 8500 as a result of the vibrations was analogous to the changes in the drag coefficient just mentioned. In a related experimental study Diana & Falco (1971) measured the drag amplification as a function of amplitude and frequency at Reynolds numbers near 25000 for a cylinder that was excited by fluid forces. A thorough bibliography of recent papers relating to the phenomenon of locking-on is given by McCroskey (1977).

Though most investigators of the phenomenon have confined their experiments to circular cylinders, Bearman & Davies (1975) and Davies (1976) have studied the effect of body shape on the locking-on. The shape of the cylinder downstream of the separation points was found to have a profound effect on the process of locking-on between the vortices and the vibrating body. However, in the case of a D-shaped cylinder with fixed separation points, changes in the base pressure, vortex strength and spacing were found which were analogous to those found previously with circular cylinders.

3. Similarity of bluff-body wakes

Both Roshko (1954*a*, 1955) and Bearman (1967) demonstrated that a unifying parameter for comparing the wakes of bluff bodies could be derived by applying relatively simple physical arguments. The result is a 'universal' wake Strouhal number St^* for vortex shedding from bluff bodies.

If one considers two shear layers a distance d' apart with the velocity just outside the layers equal to U_b, the mean velocity at separation, as shown in figure 1, then a wake Strouhal number can be defined as

$$St^* = \frac{f_s d'}{U_b} = St \frac{U}{U_b}\frac{d'}{d}. \tag{1}$$

The characteristic length d' associated with the configuration is assumed to be proportional to the ratio U_b/f_s Here the usual cylinder Strouhal number is equal to

$$St = f_s d/U, \tag{2}$$

where d is the cylinder diameter and U is the incident flow speed. By applying Bernoulli's equation to the flow just outside the boundary layer at separation, one finds the base-pressure coefficient to be

$$C_{pb} = 2(p_b - p_\infty)/\rho U^2 = 1 - (U_b/U)^2. \tag{3}$$

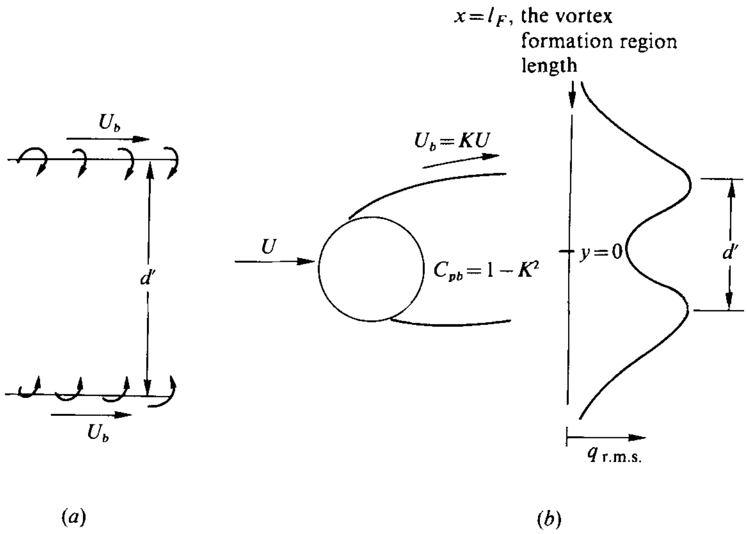


FIGURE 1. Schematic diagram of the wake. (a) Free-streamline approximation (Roshko 1954a). (b) Physical length and velocity scales for the near wake of a cylinder.

If a base-pressure parameter $K = U_b/U$ is introduced, then

$$K^2 = 1 - C_{pb} \tag{4}$$

and

$$St^* = \frac{St}{K}\frac{d'}{d}. \tag{1a}$$

A wake Reynolds number Re^* analogous in form to the universal Strouhal number St^* can also be defined by

$$Re^* = U_b d'/\nu = Re\, Kd'/d, \tag{5}$$

where $Re = Ud/\nu$ is the usual free-stream Reynolds number.

Roshko (1954b) obtained analytically the wake width d' for a circular cylinder, a 90° wedge and a flat plate by means of a so-called notched-hodograph theory. Application of this method gives the spacing of the shear layers when they become parallel, just before the roll-up into individual vortices takes place at the end of the vortex formation region. Bearman (1967), in extending Roshko's concepts, did not employ the notched-hodograph method but instead chose to determine the wake width from an analysis based on Kronauer's minimum-drag theory for vortex-street stability.† In that analysis the wake width d' was assumed to be equal to h, the downstream lateral spacing between individual vortices of opposite sign. Bearman found that $St^* = 0{\cdot}181$ for a variety of blunt-based bodies and wake interference configurations, and also obtained good agreement with a representative group of Roshko's previous findings (see figure 7).

The unifying concepts developed by Roshko and Bearman are applied in the present paper to the wakes of vibrating cylinders. As will be shown, the concept of a universal Strouhal number can be generalized to the cases of vortex-excited oscillations and forced oscillations when the vortex shedding is locked onto the vibrations. For this resonant wake–structure interaction, the wake width d' is a function of the amplitude

† A discussion of this concept may be found in the paper by Wille (1974).

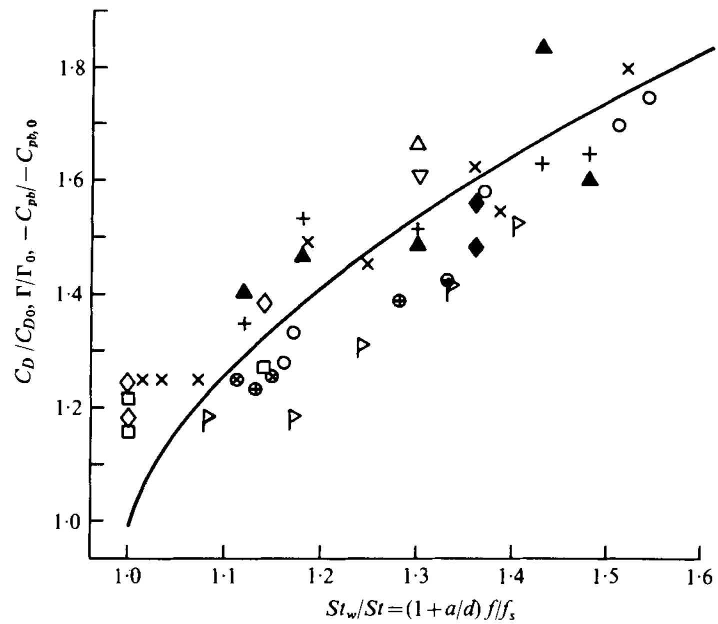


FIGURE 2. The amplification of the drag coefficient C_D, the vortex circulation Γ and the cylinder base-pressure coefficient $-C_{pb}$ during locking-on as a function of the wake Strouhal number St_w (normalized by St, the Strouhal number for the corresponding stationary cylinder). The legend for the data points is given in table 1.

and frequency of vibration and is obtained from experiment. By defining the wake width at the end of the vortex formation region, the governing length scale in the model is analogous to the original concept proposed by Roshko (1954*a*) and is easily measured. The characteristic velocity U_b and the base-pressure parameter K remain unchanged from the original model and are also dependent on the amplitude and frequency of vibration. These quantities must also be obtained from experiment.

4. Amplification of mean drag, base pressure and separation velocity

Drag, vortex circulation and base pressure

The dependence of the vortex circulation and drag on the vibration amplitude and frequency during locking-on was demonstrated previously by Griffin & Ramberg (1975) for a limited number of representative cases. A more complete view of this dependence is given in figure 2, which contains somewhat more than twice the number of data points previously reported and which has been extended to include the measurements of base-pressure amplification recently published by Stansby (1976). A complete description of the data in the figure is given in table 1. The amplification factors of the vibrating-cylinder wake properties are plotted in figure 2 as a function of the ratio of the wake Strouhal number $St_w = (a+d)f/U$ to the usual form of the stationary-cylinder Strouhal number $St = f_s d/U$ in order to take account of the range of values (St = 0·14–0·21) encountered in the results shown in the figure and listed in table 1. Such a procedure was previously used successfully by Griffin & Ramberg (1975). All of the measured points are within about 10 % of the least-squares mean line drawn on the

Symbol	Parameter	Re	St	Medium	Investigator(s)
		Flexible cylinder, forced vibration			
◇	Γ/Γ_0	570	0·21	Air	Ramberg & Griffin (1976)
□	C_D/C_{D0}	570	0·21		
△	Γ/Γ_0	450	0·21	Air	Griffin & Ramberg (1975)
▽	C_D/C_{D0}	450	0·21		
		Rigid cylinder, forced vibration			
+	Γ/Γ_0	144	0·18	Air	Griffin & Ramberg (1975)
▲	C_D/C_{D0}	144	0·18		
⊗	C_D/C_{D0}	80	0·14	Water	Tanida, Okajima & Watanabe (1973)
⊕	C_D/C_{D0}	4000	0·18		
◆	C_D/C_{D0}	7000, 10800	0·21	Water	Meyers (1975)
▷	$-C_{pb}/-C_{pb,0}$	5700	0·20	Air	Stansby (1976)
		Rigid cylinder, free vibration			
○	C_D/C_{D0}	500–900	0·21	Air	Griffin *et al.* (1973)
×	C_D/C_{D0}	21000–25000	0·19	Air	Diana & Falco (1971)
◒	C_D/C_{D0}	130	—	Water	Honji & Taneda (1968)

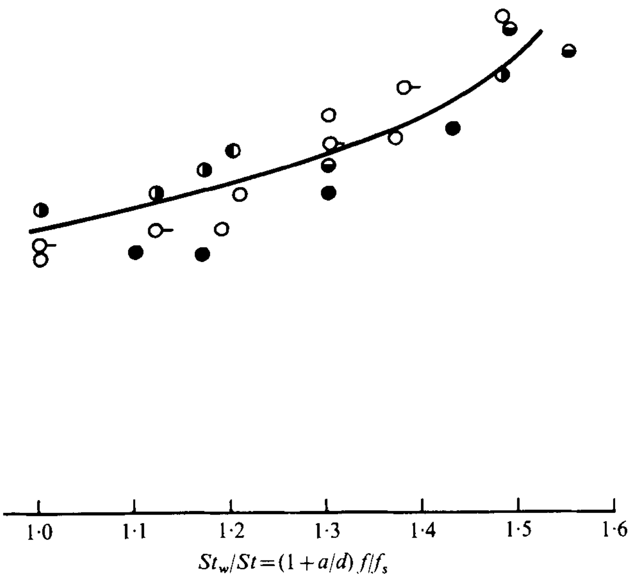
TABLE 1. Legend for the experimental measurements of the vortex circulation Γ, drag coefficient C_D and base-pressure coefficient $-C_{pb}$ in figure 2.

figure and the plotted results give a clear indication of the effects of vibration amplitude and frequency on the mean in-line fluid force, i.e. the drag coefficient C_D, and the properties of the near wake, i.e. the vortex circulation Γ and the base-pressure coefficient $-C_{pb}$.

It is important to note that the measured points refer not only to forced vibrations of a cylinder but also to resonant, flow-induced vibrations in the locking-on regime over a wide range of subcritical Reynolds numbers. The measurements by Griffin & Ramberg (1975, 1976), for instance, were made at low Reynolds numbers, $Re = 144$–570, while the base-pressure measurements by Stansby (1976) were made at a moderate Reynolds number of $Re = 5700$, both for the case of forced vibrations of a cylinder. As two additional examples, the vibrating-cylinder drag measurements reported by Griffin *et al.* (1973) were made at moderately low Reynolds numbers, $Re = 600$–800, and those of Diana & Falco (1971) were made at high subcritical Reynolds numbers, $Re = 21000$–25000. In both of the latter cases the measurements were made with cylinders which were resonantly excited by the vortex shedding. The good agreement between the various measures of the wake response, i.e. C_D, Γ or $-C_{pb}$, in figure 2 gives convincing evidence of the equivalence between the two types of resonant wake–body interaction for experiments in air and in water. The first evidence of such equivalence between forced and vortex-excited oscillation was reported by Griffin (1972), who measured changes in vortex formation and wake response for forced and freely vibrating cylinders over a range of vibration conditions at $Re = 500$–900.

Wake width

The wake width d' employed in the present formulation is defined as the lateral distance between the maxima of the velocity fluctuations at the end of the vortex formation



ɘ width d'/d, measured at the end of the vortex formation region, as a function ʋake Strouhal number St_w/St. ◒, Re = 120; ◑, Re = 144; ○, Re = 200; ○-, 350 (Griffin 1971); ●, Re = 450 (Ramberg & Griffin 1974).

This is a good measure of the mean lateral spacing of the separated function of the locking-on, as will be shown. Such a definition of the ɔeen shown to be an important scaling parameter for the wakes of ylinders over a wide range of conditions (Griffin 1971; Griffin & fin & Ramberg 1974).

h d' so obtained is plotted in figure 3 as a function of the ratio of the ımber St_w to the natural Strouhal number St. Measurements made numbers between 100 and 500 are plotted in the figure, where the ds number are minimal. The wake width d' is a function of both the e frequency of the vibrations, and the effects of both are taken into tio of St_w to St. This ratio again takes account of the different values d otherwise introduce considerable scatter into the data. In the form the effects of vibration amplitude and frequency on the wake width ined over a reasonably wide range of Reynolds numbers, Re = 100– ʋalue of $d'/d = 1{\cdot}18$, based on 22 data points for stationary circular computed from Roshko's free-streamline model results obtained at ıumbers Re^* between $1{\cdot}4 \times 10^3$ and $2{\cdot}6 \times 10^4$. This value is in good the stationary-cylinder results in figure 3, and suggests a certain een the present experimental findings and the free-streamline theory. ɘments shown in the figure were made under conditions of locking-on ɘx shedding and the vibrations, at frequencies between 80 and 120 % 'equency and amplitudes of up to 60 % of a cylinder diameter.

Investigator(s)	Re	C_D/C_{D0}§	$-C_{pb}$	K	d'/d‖	St	St^*	Re^*
Tanida *et al.* (1973)	80	1·25	1·12	1·46	1·24	0·14	0·120	145
		1·26	1·13	1·46	1·26		0·120	147
		1·58	1·42	1·56	1·37		0·123	171
Griffin & Ramberg (1974)	144	1·40	1·26	1·50	1·25	0·18	0·150	270
		1·47	1·32	1·52	1·27		0·150	278
		1·50	1·35	1·53	1·33		0·156	293
		1·84	1·66	1·63	1·38		0·155	331
		1·58	1·42	1·56	1·43		0·166	324
Ramberg & Griffin (1976)	450	1·00	0·90	1·34	1·18	0·21	0·183	711
		1·61	1·45	1·56	1·33		0·179†	934
	570	1·17	1·05	1·43	1·20		0·176	978
		1·21	1·09	1·44	1·20		0·176†	985
		1·28	1·15	1·47	1·26		0·180†	1056
Griffin *et al.* (1973)	600	1·28	1·15	1·47	1·27	0·21	0·180	1128
	660	1·33	1·20	1·48	1·27		0·181	1240
	740	1·70	1·53	1·59	1·45		0·183†	1706
Tanida *et al.* (1973)	4000	1·39	1·11	1·45	1·41	0·18	0·175	8178
		1·23	0·98	1·41	1·25		0·160	7050
Stansby (1976)‡	5700	1·19	0·95	1·40	1·23	0·20	0·175†	9820
		1·19	0·95	1·40	1·27		0·181†	10100
		1·31	1·05	1·43	1·30		0·181	10600
		1·41	1·13	1·46	1·34		0·181†	11100
		1·53	1·22	1·49	1·36		0·182†	11700
Meyers (1975)	7000	1·56	1·25	1·50	1·36	0·21	0·190	14300
	10833	1·48	1·18	1·48	1·36		0·193	21800
Diana & Falco (1971)	21400	1·50	1·20	1·48	1·39	0·19	0·178†	44000
		1·60	1·28	1·51	1·50		0·190†	48100
	22400	1·80	1·44	1·56	1·47		0·179†	51400
		1·60	1·28	1·51	1·36		0·171	46000
		1·45	1·16	1·47	1·28		0·165	42200
	24700	1·40	1·12	1·46	1·31		0·171	47200
		1·55	1·24	1·50	1·37		0·174†	50800

† Data plotted in figure 5.
‡ Stansby (1976) reported direct measurements of the base-pressure amplification $-C_{pb}/C_{pb,0}$.
§ Tabulated values of C_D/C_{D0} are taken from figure 2.
‖ Tabulated values of d'/d are taken from figure 3.

TABLE 2. The universal Strouhal number for the 'locking-on' of vortex shedding to the vibrations of circular cylinders. $-C_{pb} = -C_{pb,0}(C_D/C_{D0})$, where $-C_{pb,0} = 0{\cdot}9$ for $Re < 1000$ and $-C_{pb,0} = 0{\cdot}8$ for $Re > 1000$.

Mean velocity at separation

Upon assuming a constant time-average pressure across the separated shear layers, the average velocity U_b at separation is approximately given by

$$U_b = UK = U(1-C_{pb})^{\frac{1}{2}}, \tag{4a}$$

where the time-average base-pressure coefficient is denoted by $-C_{pb}$. Ideally one would like to have direct measurements of the base pressure for a wide range of resonant wake–structure interactions, but not all the locking-on experiments plotted in figure 2 included measurements of $-C_{pb}$ for the vibrating cylinders. However it is reasonable

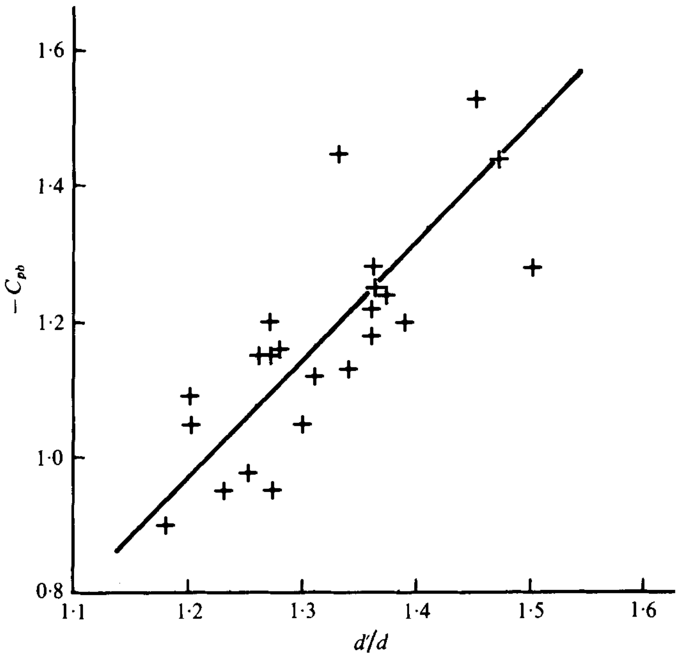


FIGURE 4. The base-pressure coefficient $-C_{pb}$ as a function of the wake width d'/d for the vibrating-cylinder experiments listed in table 2. The results plotted correspond to wake Reynolds numbers Re^* greater than 700.

to assume from the results shown in figure 2 that there is an equivalence between the amplification of the drag coefficient C_D and the base-pressure coefficient $-C_{pb}$ when the cylinder and wake are locked on in frequency. Then it is necessary only to know the base-pressure coefficient $-C_{pb,0}$ for a stationary cylinder in order to compute the base-pressure coefficient for resonantly vibrating cylinders as a function of amplitude and frequency. The base-pressure coefficient $-C_{pb}$ for the locking-on is then

$$-C_{pb} = -C_{pb,0}(C_D/C_{D0}) \tag{6}$$

and the corresponding base-pressure parameter K is

$$K = U_b/U = [1-C_{pb,0}(C_D/C_{D0})]^{\frac{1}{2}} \tag{7}$$

with C_D/C_{D0} taken from the data in figure 2. This procedure was followed with good results as shown in the next section. The values of the base-pressure parameter K obtained in this fashion are listed in table 2 with the corresponding value of the drag amplification from figure 2 and the Reynolds number. The values of $-C_{pb,0}$ for a stationary cylinder chosen for the computations were

$$C_{pb,0} = \begin{cases} 0{\cdot}9 & \text{for} \quad Re < 1000, \\ 0{\cdot}8 & \text{for} \quad Re > 1000. \end{cases}$$

These values are in good agreement with the measurements made by Roshko (1954*a*) and Stansby (1976).

The base-pressure coefficients $-C_{pb}$ obtained from figure 2 and (6) are plotted in figure 4 as a function of the wake width d'/d. A relation between $-C_{pb}$ and d'/d was

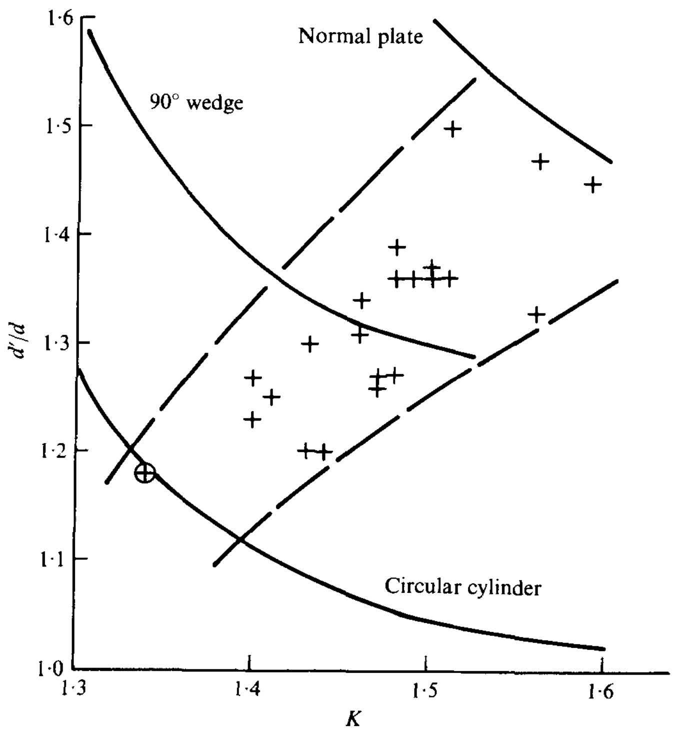


FIGURE 5. The wake width d'/d as a function of the base-pressure parameter K. +, vibrating-cylinder experiments from table 2; ⊕, stationary-cylinder experiment from table 2; ——, free-streamline theory for stationary bluff bodies (Roshko 1954*b*).

derived analytically by Roshko (1954*b*) from the notched-hodograph theory for several stationary bodies of varying bluffness. Roshko found that d'/d decreased with increasing $-C_{pb}$ (or K) for each of three cases, a circular cylinder, a 90° wedge and a normal flat plate, as shown in figure 5. This is opposite to the trend shown in figures 4 and 5 for the experimental results obtained with vibrating cylinders. It has been suggested (Roshko 1977, private communication) that any tendency for d'/d to decrease with $-C_{pb}$ is more than compensated by the increasing cylinder amplitude. The measured dependence between $-C_{pb}$ (and K) and d'/d in the present case is then a consequence of the cylinder vibrations in the locking-on regime.

The base-pressure and wake-width parameters from figures 4 and 5 are employed next in the computation of the universal Strouhal number St^* and the fraction ϵ of the shed vorticity in the fully developed, individual downstream vortices.

5. The universal Strouhal number St^*

A universal Strouhal number with which to express the important parameters relating to vortex formation in the wakes of bluff bodies of cylindrical cross-section was first proposed by Roshko (1954*a*, 1955). Later, Bearman (1967) extended Roshko's approach and devised a wake Strouhal number which was used successfully to correlate the vortex streets formed behind bluff cylinders and streamlined bodies with blunt trailing edges. Bearman's results agreed very well with measurements of St^* reported

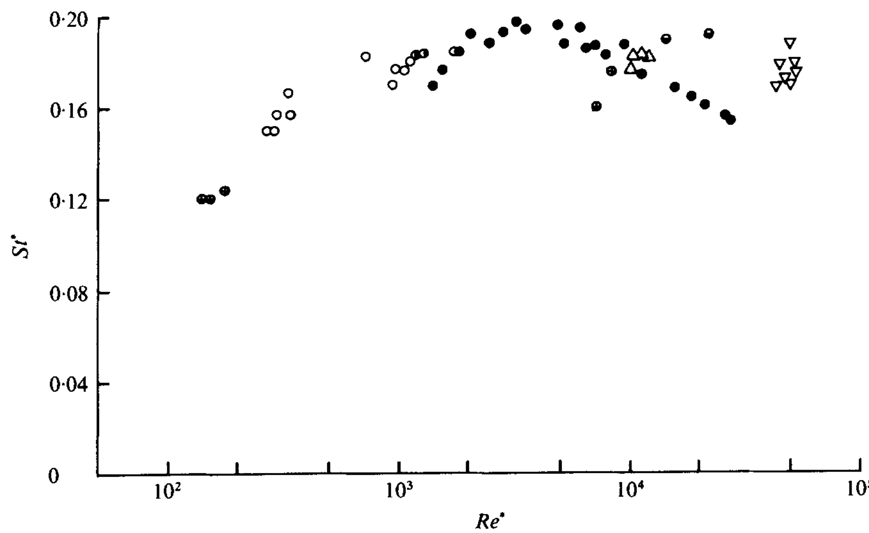


FIGURE 6. The universal Strouhal number St^* for the locking-on of vortex shedding to the vibrations of a cylinder as a function of the wake Reynolds number Re^*. Vibrating cylinders: ○, Griffin & Ramberg (1975), Ramberg & Griffin (1976), Griffin *et al.* (1973); ⊕, Tanida *et al.* (1973); △, Stansby (1976); ◒, Meyers (1975); ▽, Diana & Falco (1971). Stationary cylinder: ●, Roshko (1954*a*).

by Roshko and others. In each case some artificial means was used to interfere with the normal vortex formation process; Roshko's experiments included the attachment of wake splitter plates in various configurations and Bearman employed both splitter plates and base bleeding of low velocity air to interfere with the base flow and vortex formation.

The universal Strouhal number St^* devised for the present case of vibrating cylinders is, by analogy,

$$St^* = \frac{f_s d'}{U_b} = \frac{St}{K}\frac{d'}{d}.$$

Here the wake width d' for the vibrating cylinders is obtained from the results in figure 3 and the base-pressure parameter K from (4). The drag amplification in the equation for K is taken from the results plotted in figure 2.

The present approach differs from those of both Roshko and Bearman in that the wake width d' and the base-pressure parameter K are dependent upon the amplitude and frequency of the vibrations and are thus obtained from experiments performed under locking-on conditions. The universal Strouhal numbers St^* so obtained are listed in table 2 and are plotted in figure 6 as a function of the wake Reynolds number Re^*. It is undoubtedly an approximation to employ the values of wake width d' obtained over the somewhat limited range of Reynolds numbers shown in figure 3. However, this procedure seems justified in view of the lack of any similar results for Reynolds numbers greater than 500 and in view of the relative insensitivity of a vortex wake to Reynolds number effects once a cylinder is resonantly vibrating.

It is seen from figure 6 that the Strouhal number St^* for the wakes of vibrating cylinders is essentially constant for wake Reynolds numbers of from 700 to $4{\cdot}5 \times 10^4$.

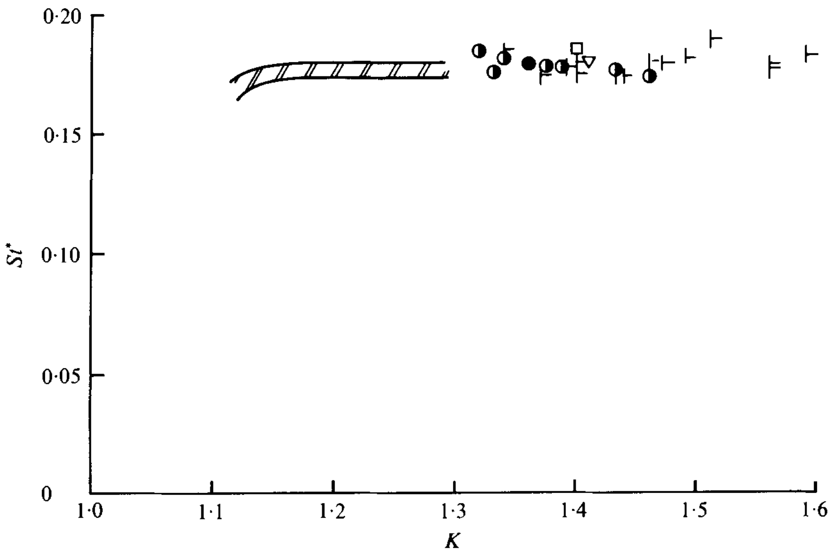


FIGURE 7. The universal Strouhal number St^* as a function of the base-pressure parameter K for various bluff-body shapes. ▽, 90° wedge (Roshko 1954*a*); □, flat plate (Roshko 1954*a*); ◑, circular cylinder, $Re = 10^2$–10^5; ●, circular cylinder, $Re = 2 \times 10^6$–10^7, Roshko (1961); ▨, mean of results from Bearman (1967); ⊢, vibrating-cylinder results from table 2. With the exception of the vibrating-cylinder results, all the data for St^* in the figure are based on the wake width d' obtained by Bearman.

At the lowest Reynolds numbers, below $Re^* = 700$, the Strouhal number is Reynolds number dependent as is usually the case. The vibrating-cylinder data in figure 6 are in good agreement with the results for stationary circular cylinders of Roshko (1954*a*), which are also plotted there. An average value of $St^* = 0{\cdot}178$ with a standard deviation $\sigma = 0{\cdot}010$ is obtained from the vibrating-cylinder results in table 2 for $Re^* > 700$. This is in excellent agreement with Bearman's finding $St^* = 0{\cdot}181$ and an average value of $St^* = 0{\cdot}180$ from Roshko's data for stationary circular cylinders.

A somewhat lower average value of $St^* = 0{\cdot}163$ was obtained by Roshko when the results for a variety of vertical flat plates with and without wake splitter plates and for a 90° wedge at wake Reynolds numbers up to $Re^* = 4 \times 10^4$ were included with those for stationary circular cylinders in figure 6. The difference between the several average values can generally be attributed (see Roshko 1955) to variations in Reynolds number, and to such history effects as the location of transition to turbulence in and the thickness of the free shear layers. Another potential influence is the particular definition of the wake width d' employed in a given formulation. While the wake width in the free-streamline theory represents the lateral distance between two idealized shear layers, the wake width in the present formulation, outlined in figure 1(*b*), corresponds to an average length scale measured at the end of the vortex formation region. The difference of about 10% between Roshko's overall average value of $St^* = 0{\cdot}163$ and the results obtained from figure 6 and table 2 is quite small and does not detract from the generality of the present findings so long as these various factors are understood.

The Strouhal number St^* can be interpreted as the ratio of two characteristic lengths: the wake width d' and the characteristic length scale given by the ratio of the separation velocity U_b and the Strouhal frequency f_s. The results in figure 6 show that a universal wake parameter such as St^* is representative of the wakes of bluff bodies not only for the conditions previously investigated by Roshko (1955), Bearman (1967) and Richter & Naudascher (1976) but also for cylinders which are forced to vibrate or are resonantly excited by fluid forces at subcritical Reynolds numbers.

A further confirmation of the universality of this concept over a wide range of conditions is given in figure 7, where St^* is plotted against the base-pressure parameter K. With the exception of the representative points for vibrating cylinders from the present paper, the results for a variety of bluff-body shapes were originally plotted by Bearman (1967). Again there is very good agreement between Bearman's and Roshko's previous findings and the present vibrating-cylinder data, which yield a constant value of St^*. This further confirms Roshko's original similarity arguments for the vortex-street wakes of bluff bodies. The vibrating-cylinder experiments discussed here clearly show the importance of the near-wake flow field and the vortex formation processes to an understanding of the fluid forces which act on resonantly vibrating structures.

6. Vorticity in the near wake

Roshko (1954*a*) has shown that the rate of vorticity generation (of each sign) in the separated regions of a bluff body is given by

$$d\Gamma/dt = \tfrac{1}{2}U_b^2(t), \tag{8}$$

where $U_b(t)$ is the instantaneous velocity at the edge of the boundary layer at separation. The average value of U_b can be closely approximated by $U_b = KU$, where again $K = (1-C_{pb})^{\frac{1}{2}}$, so that the fraction ϵ of the shed vorticity of each sign carried downstream over a shedding cycle of frequency f, by the individual vortices of circulation Γ, is

$$f\Gamma = \tfrac{1}{2}\epsilon K^2 U^2. \tag{9}$$

In the case of a vibrating cylinder when the vortex shedding is locked onto the cylinder vibrations, this equation can be written in non-dimensional terms as

$$2(f/f_s)(\Gamma/Ud)\,St = \epsilon K^2, \tag{10}$$

where f is the vibration frequency. The circulation Γ was obtained by Griffin & Ramberg (1974) and Ramberg & Griffin (1976) for a number of cases when a cylinder was vibrating. The results for Γ are plotted in figure 2, and from the corresponding values of the base-pressure parameter K from table 2 several estimates of ϵ can be obtained as a function of vibration amplitude and frequency. The resulting values of ϵ are listed in table 3 and are plotted as a function of K in figure 8.

The estimates of the circulation were obtained by matching a Kármán-vortex-street model with measured mean and r.m.s. velocity profiles in the wake (Griffin & Ramberg 1974; Ramberg & Griffin 1976). From the values of ϵ listed in table 3, it appears that for stationary cylinders slightly less than half of the shed vorticity of each sign (ϵ = 0·46–0·48) is ultimately found in the individual vortices downstream. These values of ϵ for stationary-cylinder wakes are in good agreement with the value ϵ = 0·43 obtained by Roshko (1954*a*) under similar conditions. A somewhat lower

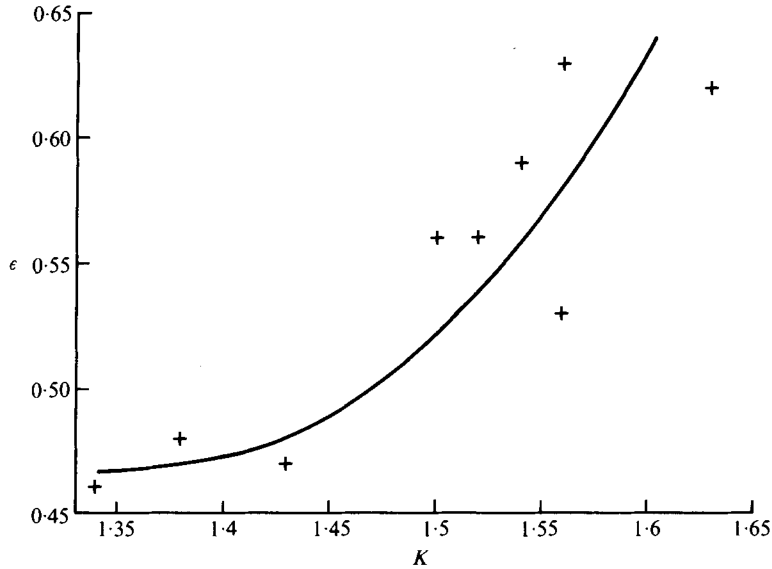


FIGURE 8. The fraction ϵ of the shed vorticity in fully developed wake vortices downstream from a vibrating cylinder as a function of the base-pressure parameter K. The legend for the data points is given in table 3.

Re	St_w	Γ/Ud†	K	ϵ
144	0·180	2·54	1·38	0·48
	0·204	3·49	1·50	0·56
	0·208	3·99	1·52	0·56
	0·231	3·86	1·53	0·59
	0·254	4·18	1·63	0·62
	0·263	4·24	1·56	0·63
450	0·208	2·01	1·34	0·46
	0·265	3·14	1·56	0·53
570	0·204	2·34	1·43	0·47
	0·233	2·76	1·47	0·48

† The circulation Γ of the individual vortices is taken from Griffin & Ramberg (1975) and Ramberg & Griffin (1976).

TABLE 3. Shed vorticity in the vortex street behind a vibrating circular cylinder. $\epsilon K^2 = 2St(\Gamma/Ud)(f/f_s)$.

value $\epsilon = 0{\cdot}26$ was measured by Davies (1976) in the case of a stationary D-shaped cylinder.

The fraction of the shed vorticity in the fully formed vortices increases with amplitude and frequency in the locking-on region until, at an amplitude of half the cylinder diameter or $K = 1{\cdot}63$, about 62–63 % of the shed vorticity is found in the fully developed vortices. This increase in ϵ is accompanied by substantial increases in the circulation of the vortices, in the drag amplification factor and in the base-pressure parameter K.

7. Summary and concluding remarks

A universal wake Strouhal number St^* has been proposed for the locking-on of vortex shedding to the vibrations of a circular cylinder. The Strouhal number so formed, $St^* = f_s d'/U_b$, is based upon the natural Strouhal frequency f_s of the incident flow, the velocity U_b at the edge of the separating boundary layer on the cylinder and the wake width d' at the end of the vortex formation region. This formulation is analogous to those previously proposed by Roshko (1954*a*, 1955) and Bearman (1967) for stationary cylinders.

A constant value of $St^* = 0{\cdot}178$ was found for subcritical wake Reynolds numbers $Re^* = U_b d'/\nu$ from 700 to 5×10^4. This finding is valid for a wide range of resonant wake–structure interaction conditions which includes both forced and vortex-excited lateral vibrations of a cylinder in the locking-on regime. Both the velocity U_b and the wake width d' are dependent upon the amplitude and frequency of the oscillations, as found from a wide range of experiments.

The universal Strouhal numbers found for vibrating cylinders are in good agreement with previous measurements made for a variety of stationary cylindrical bodies. Bearman found a constant Strouhal number $St^* = 0{\cdot}181$ for values of the base-pressure parameter $K = (1 - C_{pb})^{\frac{1}{2}}$ between 1·2 and 1·5. For vibrating cylinders in the locking-on regime, the universal Strouhal number St^* proposed here was found to be constant at $St^* = 0{\cdot}178$ for values of K between 1·3 and 1·6.

A correspondence was found between the amplification of vortex circulation, drag and base pressure as a result of lateral vibrations in the locking-on regime at subcritical Reynolds numbers from $Re = 80$ to $2{\cdot}5 \times 10^4$. For cylinder amplitudes up to a full diameter (peak to peak) and at frequencies between 70 and 120 % of the Strouhal frequency f_s, the drag and base pressure were increased by as much as 80 % from the corresponding measurements made for stationary cylinders at these subcritical Reynolds numbers.

The fraction ϵ of the shed vorticity of each sign in the fully formed individual vortices is a function of the base-pressure parameter K and of the vibration amplitude and frequency in the locking-on regime. An increase in ϵ from 0·46 to 0·63 was found as K increased from 1·34 to 1·63. This change in the fraction of shed vorticity in the downstream vortices directly corresponds to an increase in the circulation of the individual vortices from $\Gamma/Ud = 2{\cdot}34$ to 4·24 as the vibration amplitude was increased from zero to half the cylinder's diameter at Reynolds numbers between 144 and 570.

The results in the present paper have extended the concept of a universal Strouhal number for bluff-body wakes (Roshko 1954*a*, 1955) to the important case of the resonant wake–structure interactions which accompany the locking-on of vortex shedding to forced and vortex-excited vibrations of bluff bodies. Roshko, Bearman and Richter & Naudascher (1976) previously found that such a universality concept applied to the wakes of *stationary* bluff bodies of various cross-sections for various wake interference configurations and flow confinements.

This paper has been prepared as part of the research programme of the Naval Research Laboratory and is published by permission. The author is indebted to Professor Anatol Roshko for many helpful comments on the original manuscript.

Note added in proof. While this paper was in the press, Simmons (1977) reported the results of an investigation into similarities between the vortex wakes of two-dimensional axisymmetric bluff bodies. Experiments were done on several two-dimensional bodies whose separation angles α varied from 0 to 90° to the incident flow ($\alpha = 0$ refers to a D-shaped cylinder while $\alpha = 90°$ refers to a flat plate). Simmons found that a universal Strouhal number $St^* = f_s d'/U_b = (St/K)\ (d'/d)$, identical to the formulation in the present paper, was equal to 0·163 for a variety of stationary cylinders with fixed separation points, at a Reynolds number of $1{\cdot}5 \times 10^4$. It was noted by Simmons that this formulation of St^* had been suggested originally by Calvert (1967) in his work with axisymmetric, conical bluff bodies. Calvert found that $St^* = 0{\cdot}19$ for a variety of conical shapes with separation angles α between 0 and 90°.

These findings are in good ageement with the findings in the present paper, and generalize still further Roshko's original similarity concepts for the vortex wakes of bluff bodies.

REFERENCES

BEARMAN, P. W. 1967 *J. Fluid Mech.* **28**, 625–641.
BEARMAN, P. W. & DAVIES, M. E. 1975 *Proc. Conf. Wind Effects on Structures, London.*
BERGER, E. W. & WILLE, R. 1972 *Ann. Rev. Fluid Mech.* **4**, 313–340.
CALVERT. J. R. 1967 *J. Fluid Mech.* **27**, 273–289.
DAVIES, M. E. 1976 *J. Fluid Mech.* **75**, 209–231.
DIANA, G. & FALCO, M. 1971 *Meccanica* **6**, 9–22.
GRIFFIN, O. M. 1971 *Trans. A.S.M.E., J. Appl. Mech.* **38**, 729–738.
GRIFFIN, O. M. 1972 *Trans. A.S.M.E., J. Engng Indust.* **94**, 539–547.
GRIFFIN, O. M. & RAMBERG, S. E. 1974 *J. Fluid Mech.* **66**, 553–576.
GRIFFIN, O. M. & RAMBERG, S. E. 1975 *J. Fluid Mech.* **69**, 721–728.
GRIFFIN, O. M., SKOP, R. A. & KOOPMANN, G. H. 1973 *J. Sound Vib.* **31**, 235–249.
GRIFFIN, O. M. & VOTAW, C. W. 1972 *J. Fluid Mech.* **55**, 31–48.
HONJI, H. & TANEDA, S. 1968 *Rep. Res. Inst. Appl. Mech., Kyushu Univ.* **16**, 211–222.
KOOPMANN, G. H. 1967 *J. Fluid Mech.* **28**, 501–512.
McCROSKEY, W. J. 1977 *Trans. A.S.M.E., J. Fluids Engng* **99**, 8–39 (Freeman Scholar Lecture, 1976 A.S.M.E. Winter Ann. Meeting).
MEYERS, D. W. 1975 M.S. thesis, Naval Postgraduate School, Monterey.
NAUDASCHER, E. (ed.) 1974 *Flow-Induced Structural Vibrations.* Springer.
RAMBERG, S. E. & GRIFFIN, O. M. 1974 *Trans. A.S.M.E., J. Fluids Engng* **96**, 317–322.
RAMBERG, S. E. & GRIFFIN, O. M. 1976 *Naval Res. Lab. Wash. Rep.* no. 7945.
RICHTER, A. & NAUDASCHER, E. 1976 *J. Fluid Mech.* **76**, 561–576.
ROSHKO, A. 1954*a* *N.A.C.A. Tech. Note* no. 3169.
ROSHKO, A. 1954*b* *N.A.C.A. Tech. Note* no. 3168.
ROSHKO, A. 1955 *J. Aero. Sci.* **22**, 124–132.
ROSHKO, A. 1961 *J. Fluid Mech.* **10**, 345–356.
SIMMONS, J. E. L. 1977 *Aero. Quart.* **28**, 15–20.
STANSBY, P. K. 1976 *Proc. A.S.C.E., J. Engng Mech.* **104**, 591–600.
TANIDA, Y., OKAJIMA, A. & WATANABE, Y. 1973 *J. Fluid Mech.* **61**, 769–784.
TOEBES, G. H. 1969 *Trans. A.S.M.E., J. Basic Engng* **91**, 493–505.
WILLE, R. 1974 In *Flow-Induced Structural Vibrations* (ed. E. Naudascher), pp. 1–16. Springer.